Orbital Perturbations and Stationkeeping of Communication Satellites

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I. Introduction

SINCE the success of SYNCOM in 1963, the world has witnessed a phenomenal growth in the field of communication via satellites. Sir Arthur Clark, who conceived of communication satellites over 30 years ago, once said[1] "...what we are building now is the nervous system of mankind which will link together the whole human race, for better or worse, in a unity which no earlier age could have imagined..." In the words of the late Vikram Sarabhai,[2] "...several uses of outer space can be of immense benefit to developing nations wishing to advance economically and socially. Indeed without them it is difficult to see how they can hold on their own in a shrinking world..." The global impact of the communication satellite technology is evident from the increasing number of specialist conferences[3-6] and sessions on the systems and their applications. A number of studies[7-12] emphasize the potentials and problems of satellite communication for developing countries in particular. With the experience of INTELSAT's, Moliniyas, and a number of successful experiments like SITE in India using ATS-6, several countries (advanced as well as developing) have launched, acquired or proposed satellites for domestic and regional services.[13-20] Table 1[21] lists the number of geosynchronous satellites launched or planned to be launched by various countries and agencies during 1963-1980. Of these 93 satellites, 77 are communications satellites of various types.

Table 1 Satellites launched or planned during 1963-1980

Sponsor	Satellites
Canada	5
Europe	6
France/West Germany	2
India	1
Indonesia	2
Italy	1
Japan	5
NATO	3
United Kingdom	2
U.S.	50
USSR	15
West Germany	1

The applications of communication satellites at present include national and international telecommunication, radio and TV broadcast, defense communication, aircraft and ship navigation, computer interconnections and data transmission, meteorological observations and warning, satellite tracking, health and educational services using TV, etc., It is interesting to note that, although the U.S. is thinking of employing the satellites for interactive broadcast enabling personalized use,[22] the Europeans[23] and Japanese wish to employ them for conventional broadcast for entertainment and education. The developing countries like Brazil,[24] India,[25] and Indonesia,[26] on the other hand, are hoping to use the systems for mass education. With the growing perfection in the technology and the decreasing annual cost per communication channel, much more interest in the systems can be anticipated throughout the world. The Space Shuttles now under testing will reduce the launch cost drastically.[27] Simultaneous to these developments, the increasing interest in the concept of geosynchronous solar power stations[28,29] should also be noticed.

The growth of the technology has naturally been coupled with the increasing complexity of the spacecrafts and simplification of the ground receivers. To enable small fixed ground antennas to receive signals of sufficient strength, satellites with highly directional antennas are being used or designed, imposing very close tolerances on the permissible drift from the preferred location and orientation. Stringent international regulations are also imposed to avoid interference with other systems.[30] As against SYNCOM's, the first experimental geostationary satellites, which were allowed to drift by several degrees under the action of various perturbations, most satellites today are required to maintain a station accuracy of better than 0.1 deg. This is achieved by incorporating suitable thrusters aboard the spacecraft. As the spacecraft lifetime is essentially limited by the available fuel, it is imperative to design optimal schemes for orbital control.

The system performance and cost-effectiveness depend upon several interlinked elements, such as space segment, ground station, control center, and computing center. As stated previously, the design of the ground-station antenna system imposes a direct restriction on the allowable drifts of the satellites, thus having an impact on the design of the stationkeeping system.[31] The accuracy of stationkeeping depends upon the ability of orbit determination, which must be at least one order of magnitude better.[32] This makes

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proper mathematical modeling and efficient algorithms essential. Compact analytic and semianalytic theories of motion are extremely helpful in reducing the demands on the computers, and thus enabling utilization of simpler ground stations.

Although the roots of the analysis date back nearly two centuries, much of the vast literature related to orbital perturbations and stationkeeping systems and logic has appeared in the past 15 years. The designers and engineers find themselves rather overwhelmed. The pressures of time schedules often make it impossible for them to keep track of analytical developments, sometimes even of the directly useful ones. This papers makes a modest attempt to survey some of the existing literature on the subject with the dual objective of providing the designers with a list of ready references and briefing the beginners and nonspecialists on the exciting problems and challenges involved. The specialist, of course, will recognize this as only the tip of an iceberg. We begin with the discussion of orbital perturbations.

II. Orbital Perturbations

A terrestrial satellite is said to be synchronous if it travels with the angular velocity matching the Earth's spin and thus remains at the same meridian continuously. A synchronous satellite in the equatorial plane, under ideal conditions, also rigorously maintains its latitude, and is called geostationary.[33] The ideal conditions connote a dynamic system of two bodies which can be represented by point masses. However, the reality is far from this. The orbit is influenced by many causes, such as 1) anisotropic terrestrial potential; 2) gravitational influences of other bodies, particularly those of the sun and moon; 3) pressure due to direct solar radiation, Earth's radiation and albedo, and satellite's self-radiation; 4) acquisition and reacquisition errors; 5) leakages from spacecraft; 6) coupling with the attitude dynamics and controllers; 7) interaction with charged environment, cosmic rays, Earth's magnetic field, etc; and 8) relativistic effects. Of these, the literature suggests the following to be of greater significance for geostationary satellites[34-36]: a) anisotropic geopotential, 2) luni-solar attraction, and 3) solar radiation pressure. Briefly, the Earth's gravity field and radiation pressure give rise to mainly in-plane perturbations, whereas the Sun and Moon generate out-of-plane perturbations. The Sun and Moon induce a long-term variation of inclination and node. Luni-solar in-track perturbations are rather small. Radiation pressure causes an annual variation of the eccentricity. The ellipticity of the equator generates long-period variation of semimajor axis and longitude. Finally, oblateness of the Earth and luni-solar effects slightly modify the synchronous radius of a geostationary orbit.[35] The following survey considers the three perturbative sources in some detail.

A. Anisotropic Geopotential Effects

Orbital perturbations attributed to the anisotropy of geopotential is one of the several problems confronted by astrodynamicists from the very genesis of the subject. The potential of an isotropic sphere with homogeneous concentric layers depends on radial distance alone. However, for triaxial Earth, the potential depends on longitude and latitude as well. The mathematical terms representing the difference between some approximation to the geoid and a sphere are customarily called "harmonics." (The mean sea-level surface extended through the continents is often referred to as the geoid.) The "zonal harmonics" describe the deviation of a meridian from a circle, whereas "tesseral harmonics" indicate the deviation of a latitude contour from a circle. Using these, the Earth's potential V at a given point is[37,38]

$$V=-\frac{\mu}{r}\Big[1+\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^{n}P_{nm}(\cos\theta)\times(C_{nm}\cos m\lambda+S_{nm}\sin m\lambda)\Big] \tag{1}$$

where r is geocentric distance, λ is geographic longitude, θ is polar angle, μ is gravitational constant times mass of the Earth, R is the mean equatorial radius of the Earth, C_{nm} and S_{nm} are numerical coefficients characterizing the Earth's mass distribution, and P_{nm} is the associated Legendre polynomial. With the terms independent of longitude separated out, Eq. (1) takes the form

$$V=-\frac{\mu}{r}\Big[1-\sum_{n=2}^{\infty}J_n\left(\frac{R}{r}\right)^{n}P_n(\cos\theta)+\sum_{n=2}^{\infty}\sum_{m=1}^{n}J_{nm}\left(\frac{R}{r}\right)^{n}P_{nm}(\cos\theta)\cos m(\lambda-\lambda_{nm})\Big] \tag{2}$$

where

$$J_{nm}=(C_{nm}^2+S_{nm}^2)^{1/2} \tag{3a}$$

$$m\lambda_{nm}=\arctan(S_{nm}/C_{nm}) \tag{3b}$$

Here $J_{21}=0$, $J_2=1.083\times10^{-3}$, and all other coefficients are of an order of magnitude 10^{-6} or higher. Using the resonant orbit observations, terrestrial tests, and extensive analysis,[39-43] the coefficients in the preceding expressions have been determined to a very good degree of accuracy.[44-48] The latest model[48] gives values up to $n=22$, $m=14$.

Early studies concerned with the influence of the Earth's nonsphericity on the geostationary satellites employ linearized models. Using polar coordinates, Frick and Garber[49,50] and Perkins[51] present the basic picture of orbit perturbations of synchronous satellites. Blitzer et al.[52,53] find solution of the linearized equations in Earth-fixed coordinates. It is established that points on the geostationary orbit at the extensions of the Earth's minor axis correspond to the stable equilibrium positions, whereas those at the extremities of the major axis are unstable. Allan[54] employs the energy relationship of two-body motion to reveal the influence of equatorial ellipticity on various orbital elements. A semianalytical variational approach is adopted by Musen.[55] Wagner[56,57] considers the influence of geopotential through the fourth order and concludes that earlier results are modified only in detail. He constructs "stable regions" within which the drifts are self-limiting. The solution is used[58] to establish to a better precision the Earth's equatorial ellipticity using the observations of SYNCOM II. Blitzer[37,59] derives a closed-form expression for equilibrium longitudes as

$$\lambda_0=\lambda_{22}+\frac{s\pi}{2}-\frac{\sum\sum_{(n-m,\ \text{even})}(mJ_{nm}A_{nm}/z_0^n)\sin m(\lambda_{22}-\lambda_{nm}+s\pi/2)}{\sum\sum_{(n-m,\ \text{even})}(m^2J_{nm}A_{nm}/z_0^2)\cos m(\lambda_{22}-\lambda_{nm}+s\pi/2)} \tag{4}$$

where

$$A_{nm}=\frac{(-1)^{(n-m)/2}(n+m)!}{2^n[(n-m)/2]![(n+m)/2]!} \tag{5a}$$

$$z_0=(r/R) \tag{5b}$$

$$s=0,1,2,3 \tag{5c}$$

λ_0 for $s=1,3$ only are stable. Cambi[60] employs Liapunov criteria to establish the stability.

Geosynchronous satellites exhibit resonance characteristics resulting from various harmonics of the geopotential.[61,62] Morando[63,64] employs a modified von Zeipel procedure to obtain the stable positions and the period of librations of the tesseral harmonics separately. Musen and Bailie[65] generalize the earlier work of Musen[55] to eccentric inclined orbits and obtain conditions of stability for synchronous orbits employing canonical elements and Hamilton-Jacobi partial differential equations. Allan[66-68] derives expressions for time rate of all orbital elements due to tesseral harmonics and employs[69] them to analyze the motion of SYNCOM II and III to deduce the longitudinal dependence of the geopotential.

More recently, Zee[70] analyzes near-synchronous orbits of small inclination and eccentricity, in a spherical coordinate frame, employing asymptotic method in nonlinear mechanics. Closed-form solution for an improved second-order approximation are obtained. Hanavan[71] feels that this field has been dominated unnecessarily by general perturbation methods and methods in nonlinear mechanics. He resorts to the method of variation of parameter, with step-by-step numerical integration. Although his results are interesting, their utility is questionable because of a mistake in the computation of "stable position." Flury[35,72,73] arrives at many useful results, included below, through a first-order perturbation theory. Vashkovjak[74,75] derives approximate expressions for the variation of parameters due to various perturbative sources. His semianalytical approach yields results of good accuracy. Richardson[36] analyzes long-period behavior of the Keplarian elements using the extended phase-space canonical formalism.

Some of the important findings from the studies just mentioned are summarized as follows:

1) The relationship between the mean longitude of the satellite measured from the minor axis of the Earth's equator and the time is analogous to that of the motion of a simple pendulum.

2) For position around the minor axis, the satellite oscillates with an amplitude equal to the angle between the initial position and minor axis, and contains both diurnal and long-period components. Accounting for only the resonant part of the J_{22} term, the period is given by[35]

$$T = \frac{2a_s}{3\sqrt{-J_{22}}\, w_e a_e} F\left(\sin\lambda_{\max}, \frac{\pi}{2}\right) \tag{6}$$

where a_s, a_e denote the semimajor axis of the satellite and the Earth's elliptic equator; w_e is the Earth's angular rate; F is the elliptic integral of the first kind; and $\lambda_{\max}$ is the maximum deviation from the nearest stable point. The expression yields the minimum period to be 841 days. Similarly, the maximum amplitude of long-period perturbation of the semimajor axis is about 33 km.

3) The drift rate is maximum for positions 45 deg away from the stable positions.

4) The orbital characteristics such as regression of the line of nodes, the rotation of the line of apsides, orbit inclination, semilatus rectum, and eccentricity are subjected to similar long-term variations.

5) The J_{20} term causes secular perturbations in the longitude of the ascending node, the argument of perigee, and the mean longitude. As a consequence, the second zonal harmonic modifies the synchronous radius. The short-period perturbations (period = 1 day) have the following amplitude[35]: semimajor axis ≤ 1 m, eccentricity $\sim 4\times10^{-5}$, inclination (for $i\sim0.4$ deg) $\sim7\times10^{-6}$ deg, and mean longitude (for $i\sim0.4$ deg) $\sim3\times10^{-8}$ deg.

6) Harmonics other than J_{22} may contribute about 15% to the tangential perturbing force. A subset of 6×6 field in the gravitational potential is sufficient for practical needs.[74]

Figure 1 (extracted from Ref. 76) presents a broad picture of the motion in the equatorial plane.

B. Luni-Solar Attraction

The presence of the Moon and to a lesser extent that of the Sun result in significant perturbations of geosynchronous satellites. The main effect is the motion of the orbital plane, which is a combination of the rates of precession about three axes: the Earth axis, the normal to the lunar orbit, and the normal to the ecliptic.

The equation of perturbed motion can be obtained from the n-body relations given by (with c.m. of the system as origin)[38]

$$m_i \frac{d^2 q_i}{dt^2} = \frac{\partial \phi}{\partial q_i} \qquad (q_i = x,y,z)$$

where

$$\phi = \frac{k^2}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{m_i m_j}{r_{ij}} \qquad (i \neq j) \tag{7}$$

k is the Gaussian constant, m_i is the mass of ith body, and r_{ij} is the distance between the ith and jth bodies.

The analysis of luni-solar perturbations on artificial satellites finds its roots in the classical work on the lunar theory. The nineteenth century work of Cayley[77] and Hansen,[78,79] for example, forms the basis of many current studies. Of course, since the orbital eccentricity and inclination with respect to the plane of the perturbing body can be large in the case of artificial satellites, new solutions are also essential. In a pioneering study, Kozai[80] showed a possibility of drastic reduction in the lifetime due to luni-solar attraction on artificial satellites in highly eccentric orbits. This was followed by a large number of investigations, notably those by Moe,[81] Musen,[82-84] and Lidov.[85] Musen's semianalytic approach is suitable for numerical integration and converges for all eccentricities and inclinations.

One of the first studies on the luni-solar perturbations of synchronour satellites is by Sehnal,[86] who investigates the individual effects of various perturbing terms using a numerical approach. Allan[87] finds the rate of variation of orbital elements. In a later study with Cook, he[88] considers long-period motion of the plane of the synchronous orbit. They show that the pole of an initially equatorial synchronous orbit performs a precessional motion with a period of about 52 years. The inclination reaches a maximum of about 15 deg. The situation is illustrated by Fig. 2. Frick and Garber,[49] through a linearized analysis, show that luni-solar attractions result in small oscillatory in-plane deviations, with a maximum possible value of 45 miles. The principal effect is to produce a change in the orbital inclination at a rate of about 0.85 degrees per year. In a sequel, Frick[89] generalizes the study to near-circular inclined orbits. Morando[90] shows that, although the libratory character of the motion is not altered by luni-solar perturbations, equilibrium nevertheless is made impossible. The orbital stability of two eccentric 24-h

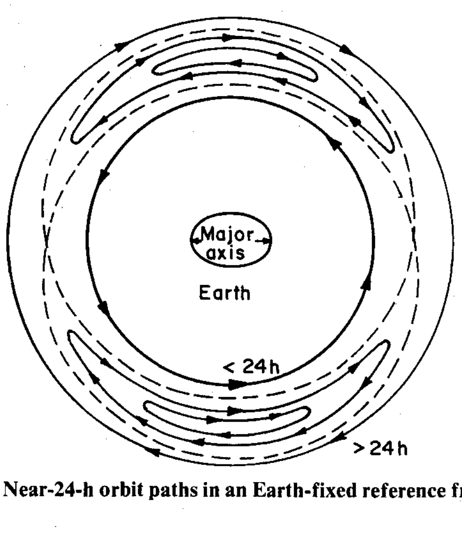

Fig. 1 Near-24-h orbit paths in an Earth-fixed reference frame.[76]

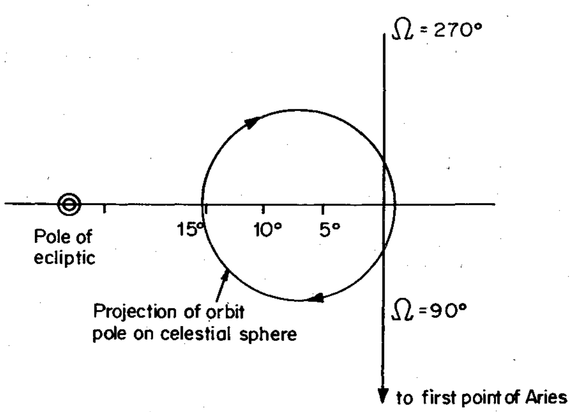

Fig. 2 Motion of the pole of a synchronous orbit.[35,88]

satellites subjected to the lunar and solar perturbations is studied by Martyenko,[91] who finds higher-eccentricity orbits to be more susceptible to the perturbations. Nita[92] finds small periodic displacements. Zee[93] extends his earlier work[70] to analyze the effect of luni-solar attraction. He employs the asymptotic method and monofrequency oscillation approach in nonlinear mechanics. Assuming the Earth, Sun, and Moon to be coplanar and moving in circular paths, he concludes that, for initially equatorial synchronous satellite, 1) the orbital plane rotates by 14 deg 43 min in 26.8 years, 2) initial rate is 0.86 deg per year, 3) orbital plane regresses by 1.595×10^{-4} rad per day, and line of apsides advances by the same rate, and 4) eccentricity grows, even for very small initial values. For eccentric ($e = 0.1$), inclined ($i = 10$ deg), synchronous orbits, useful for defense satellites, Hanavan[71] concludes, for a satellite at an initial location of 60°E, the following secular effects over 1 year ($t_0 = 2{,}441{,}317$ Julian date): 1) regression of line of nodes, 3.4 deg; 2) advance of perigee, 1 deg; 3) decrease in inclination, 0.2 deg; 4) decrease in eccentricity, 0.0004; and 5) increase in semimajor axis, 0.0003 geocentric units. Flury,[72] through a first-order perturbation analysis, shows the resonance frequencies to be modified slightly. For a specific case, he finds the secular and periodic perturbations to be in close agreement with the earlier studies. Vashkovjak[74,75] includes the luni-solar effects in his semianalytic approach. Richardson[36] refines the earlier work by taking a second-order approximation of the Moon and accounting for the Sun's elliptic motion about the Earth-Moon barycenter.

A few studies demonstrate utilization of the perturbing forces to advantage. Billik[94] shows that, for a 24-h near-equatorial circular satellite orbits, considerable cross-track sustaining velocity (North-South stationkeeping effort) can be saved by placing the satellite initially in the maximum allowable orbital inclination (rather than in the nominal equatorial plane) with an appropriate orbit ascending node location such that the inclination decreases to zero. This has been done successfully in a few satellites. In a recent study, Kamel and Tibbitts[95] establish the relative stability of an orbit with the ascending node at the vernal equinox and inclination of approximately 7.5 deg. Here there is a balance between the precession about the Earth's pole due to oblateness and the luni-solar precession, averaged over the 18.6-year cycle of the Moon's ascending node, about the pole of the ecliptic. There is an approximately 0.5-deg forced fluctuation with period 18.6 years and a free oscillation with period 52 years.

It may be noted that the effort involved in overcoming the small North-South drifts due to the luni-solar attraction is generally much more than that for East-West drifts, resulting from the Earth's triaxility. A number of satellites, because of energy limitations, have been allowed to drift in a North-South direction. With the present stringencies, of course, this may not be possible for long-life missions.

C. Solar Radiation Pressure Effects

With the increasing demands on the geosynchronous satellites under the limitations of launch capabilities, their complexity of configuration and sizes are growing rapidly. ATS-6 and CTS are just a beginning of such systems. The geosynchronous satellite solar power station (SSPS) [28] several square kilometers in size, is an example of things to come. For such spacecraft with high area-to-mass ratio, the solar radiation pressure (SRP) will probably be the most predominant source of perturbation. The need for investigation of the effects was realized quite early when the motion of Vanguard 1 could not be explained fully on the basis of the Earth's oblateness and the luni-solar attraction. The orbital performance of the balloon satellite Echo gave a further impetus to such studies.

The force due to the SRP on a satellite can be found by integrating over the entire area[96] the force acting on an elemental area dA given by

$$d\bar{F} = -P_0 dA \, |\cos\alpha| \, \{ (1-\tau)\bar{u}_s + \rho\bar{u}_r \} \tag{8}$$

where

P_0 = SRP for normal incidence (4.65×10^{-6} N/m^2)
α = angle of incidence
$\bar{u}_s$ = unit vector along the Sun rays
$\bar{u}_r$ = unit vector along the reflection direction
τ, ρ = surface transmissivity and reflectivity, respectively

The early development of the theory of SRP perturbations can be attributed to Musen,[97] Bryant,[98] Kozai,[99] Parkinson et al.,[100] and Shapiro and Jones,[101] who analyzed the performance of Vanguard 1 and the balloon satellite Echo. The resonance phenomena, under which the perturbations are enhanced considerably, identified by Musen[97] and Shapiro and Jones,[101] are considered in detail by Polyakhova,[102] neglecting the Earth's shadow. Double-resonance cases are established by Hori,[103] using the method of successive approximation. In general, under resonance conditions, the long-period terms appear in the formulas for all orbital elements except the semimajor axis, which is subjected to short-period perturbations only. This can cause a significant reduction in the lifetime of a satellite. The SRP perturbations are influenced both by orbital precession due to oblateness and by the Earth's motion around the Sun. For $i < 50$ deg, the two effects are compensatory.[101]

The inclusion of shadow effects introduces short-period variations and discontinuities, and the analysis becomes involved. Some special cases are considered by various authors. When perigee lies in a line perpendicular to the Earth-Sun line and the orbit is ecliptic, the orbit period varies most. If the shadow is asymmetric to the line of nodes, strong variation of longitude of the line of nodes results. For the case of maximum asymmetry of shadow normal to the line of nodes, the inclination of the orbital plane is perturbed strongly. Generally, the eccentricity and longitude of perigee are the most-affected elements.[99,100] Lala and Sehnal[104-106] treat the shadow as a special mathematical function in the perturbation equation and analyze the behavior of satellites such as Echo, Explorer 19, and PAGEOS through a semianalytic approach. The analysis of perturbation through a progression of simplified models is advocated by Levin,[107] whereas Mello[108] successfully employs the von Zeipal method in conjunction with canonical equations. In a recent study, Patterson and Kissell[109] compare theoretical predictions and observed accelerations of the PAGEOS satellite, whose behavior leaves many questions unanswered. Vashkoviak[110] proposes an analytical method to determine orbital disturbances of a balloon satellite subjected to SRP, Earth's oblateness, and shadow regions.

The particular case of SRP perturbations of synchronous satellites has begun receiving attention relatively recently. Lubowe[111] considers large disk-shaped and lenticular satellites in synchronous orbits subjected to the perturbation due to Earth's elliptic equator and SRP. Through a step-by-step numerical integration, he concludes that the satellites either execute bounded oscillation around a stable point, drift away from it, or blow away from the Earth, depending upon a parameter (related to A/m and reflectivity). Perrine,[112] through an analog simulation, finds that, qualitatively, a large reflecting mirror continuously shifts in a direction normal to the Earth-Sun line. Their results are in close agreement with those of Polyakhova.[102] Zee[113] presents a comprehensive picture of the orbital perturbations of geostationary satellites subjected to luni-solar attractions and SRP. Assuming the solar panel to continually lie normal to the Sun-satellite radius vector and neglecting the Earth's shadow effects, he concludes that the SRP induces eccentricity varying from zero to its maximum value and back to zero in 1 year. The rate of rotation of the line of apsides is approximately half of that of the rotation of the Earth around the Sun. Although the semilatus rectum remains practically invarient during 1 year, the semimajor axis reaches a maximum value at the midyear and then reduces to its starting value by the end of the year. Flury[35] gives closed-form expressions for long-term variation of the semimajor axis and eccentricity and finds, for a satellite with A/m of 0.1 m^2/kg, the maximum perturbations as $\delta a = 350$ m, $\delta e = 0.002$ for 1 year period, and $\delta i = 8 \times 10^{-5}$ deg for 6 months. Modi et al.[114] analyze the motion of the Canadian Telecommunication Satellite (CTS) which has very large solar panels, under the influence of SRP and conclude findings similar to those just discussed. In a more recent study, Vander Ha and Modi[115] examine the motion of satellites in the ecliptic orbits subjected to SRP. The solution, obtained using the two-variable expansion procedure, is found to be in close agreement with the numerical solution. CTS and SSPS classes of satellites are considered as illustrations.

It may be noted that the geostationary statellites are shadowed by the Earth for a maximum of 70 minutes per day at the two equinox points. This reduces to zero at about ± 22½ days away from them.[116] Thus the total shadowed region per year is less than 5%. This justifies omission of shadow effects in most of the preceding studies. However, the perturbation in the mean anomaly due to shadowing can be significant even for geostationary satellites.

D. Numerical Solutions

A discussion on orbital perturbation remains incomplete without a mention of numerical approaches of solution, which inevitably support special perturbation methods. Also, although the analytical formulation of a perturbation problem may be elegant, numerical results may be meaningless if the technique is not accurate enough.[117]

In the past two decades, numerous generalized and special-purpose algorithms for orbit determination and prediction have been developed.[118] It is beyond the scope of the present survey to present a comprehensive review of these methods. However a few recent references,[6,31,32,36,38,48,71,74,118-126] with an emphasis on geosynchronous satellites which may be of interest to the readers, are included here. A special mention must be made of Ref. 126, which gives a comprehensive summary of the numerical approaches used by Goddard Space Flight Center, NASA, and Refs. 31 and 32 which show that, by dual-ranging and having four to eight data sets per orbit in conjunction with a good mathematical model, it has been possible to achieve an orbit determination accuracy of better than the specified value of 0.005 deg for TELESAT The readers may also find the extensive reviews on the analytical and numerical methods in satellite theory and celestial mechanics by Chebotarev,[127] Dallas,[128] Abalkin et al.,[129] and Marchal[130] useful.

III. Stationkeeping

As stated in the beginning, with the increasing use of communication satellites, the trend is toward development of more powerful, high-capacity spacecraft with highly directional antennas operating with simple fixed-antenna ground stations. This puts a rather exacting demand on the satellites to maintain their relative position and orientation within close tolerances. The dominant perturbations of a synchronous orbit, discussed previously, consist of terms with periods from half a day up to several years, and secular terms. Depending upon the mission requirements, one designs a stationkeeping system to overcome them.

Basically, orbit correction involves adding or subtracting a velocity increment or ΔV along a predetermined vector, either by applying small-thrust impulse bits intermittently or continuously over a prolonged period or by applying a single major pulse at the optimum instant. Depending on the nature of orbit errors, ΔV might be applied both in-plane of the orbit or out-of-plane.[131] The choice of means to provide this is governed by many factors, such as duration of the mission, required degree of control, kinds of magnitude of perturbations, system configuration, control force requirements, mass momentum and energy requirements, propulsion system characteristics, compatibility, and (above all) reliability.[132] Being coupled and complimentary, it is preferable to acheive orbital correction and attitude control using one total system.[33,133-135] However, mainly because of the different levels of requirements, attitude-control systems and orbit-correction systems mostly have been developed separately.

In the past 15 years, many types of thrusters (chemical and electrical) have been suggested, developed, and successfully employed on several operational satellites. Many more are in the offing. A feasibility of using the solar radiation pressure to advantage has also been thought of. The cost of space missions being really astronomical, any saving is welcome. It is hardly surprising, therefore, to find the amount of literature on optimal control. In the following sections, some of the literature on these aspects is surveyed briefly.

A. Active Orbital Control Systems

The control thrusters available today can be broadly classified as chemical or electric. Among chemical systems, the following have been developed and used for various applications[131,134,136]: cold gas (e.g., nitrogen), monopropellant (hydrazine), liquid bipropellant (N_2O_4-UDMH), cryogenic bipropellant (LOX-hydrogen), hybrid (LOX-butylate), and solid (proplyl nitrate). The electric thrusters fall under four major classes:[137] electrothermal (e.g., ammonia, hydrazine), plasma (teflon pulsed thruster), colloid (doped gylcerol), and ion thrusters (cesium, mercury).

Numerous excellent surveys on the historical development, present status, and future programs in various countries on these systems have appeared in the literature recently.[137-146] As such, we shall not dwell upon the systems here. Interested readers should find the survey by Clark,[137] in particular, very useful.

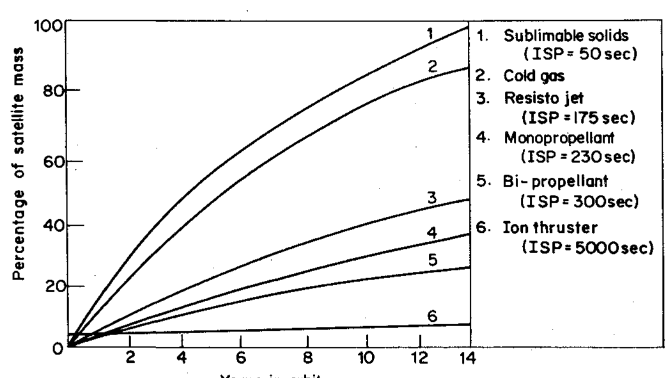

Fig. 3 Mass of orbit-correction system of a satellite for various types of propulsion.[147]

The applicability, constraints, and comparative study of the foregoing systems for stationkeeping of geostationary satellites is presented in a number of studies.[131-134,140,147-149] Figure 3, extracted from Ref. 147, compares the variation of masses of several orbit-control systems as a percentage of the satellite mass with the satellite lifetime. It is generally felt that for moderate lifetimes electrolyzed water and hydrazine are superior, from several considerations. For larger, extremely fragile communication satellites of the current and future generation, requiring precision, long life, and low weight, pulsed-plasma and ion thrusters may provide the answer. The high power requirement and low efficiency are the factors limiting their usage. Also, for initial acquisition, another more powerful system is needed.[150] In a recent study, Bassner et al.[151] suggest a hybrid system employing ion propulsion for North-South stationkeeping and chemical bipropellant system for East-West control, attitude control, and initial acquisition. For long-life missions, such a system appears to be more promising.

Of the many systems suggested and developed, some have been proven for their prolonged operation in geosynchronous orbits. The basic design aspects and their performance is discussed by several authors.[26,32,133,137,152-165] Table 2, extracted from these studies and Ref. 166, lists a number of geosynchronous satellites and their statonkeeping systems.

The list in Table 2 indicates a general preference for monopropellant hydrazine system and pulsed-plasma systems. A large number of satellites being spin/dual-spin stabilized, pulsed-jet systems are adopted. Now, however, there is a trend toward three-axis stabilized systems. For such satellites, continuous or intermittently continuous microthrusters also should become useful.

Table 2 Geosynchronous satellites and stationkeeping systems

Satellite	System
SYNCOM I, II, III	Hydrogen peroxide; nitrogen
INTELSAT I, II	Hydrogen peroxide
INTELSAT III, IV, IVa	Hydrazine
INTELSAT V[a]	Hydrazine
ATS-1	Hydrogen peroxide
ATS-3	Hydrogen peroxide, hydrazine
ATS-5	Hydrazine, ammonia resistojet
ATS-6	Hydrazine
CTS (Canada)	Hydrazine
SYMPHONIE (French/German)	Hydrazine (bipropellant)
ANIK 1, 2 (Canada)	Hydrazine
PALAPA 1, 2 (Indonesia)	Hydrazine
WEBSTER[a]	Hydrazine
OTS(ESA)	Hydrazine
RCA GLOBCOM[a]	Hydrazine
TVBS (Europe)[a]	Mercury ion thruster for N-S
APPLE (India)[a]	Hydrazine
SMS[a]	Pulsed-plasma thruster
MAROTS (ESA)[a]	Hydrazine
TACSAT	Hydrogen peroxide
SKYNET I, II (Britain)[a]	Hydrazine
NATO II	Hydrazine
DSCS II	Hydrazine
LES-6	Ammonia, pulsed-plasma (autonomous)
LES-8,9	Pulsed-plasma thruster (autonomous)
FLTSATCON/AFSATCON[a]	Hydrazine

[a] Proposed.

B. Active Orbital Control Analysis

Analytical studies on active stationkeeping have kept pace with the development of systems. The investigations can be divided into two broad groups, namely, 1) feasibility studies, design of control laws, and pre- and postlaunch performance analysis; and 2) optimization studies.

Some of the earliest studies in the first group are by Roberson,[167] Olds,[132] Kang and Kenehan,[168] Earl and Day,[134] and Neufeld and Anzel,[169] who find the velocity requirements to remove errors introduced by various perturbing forces. Table 3, extracted from Ref. 147 and after minor adaptation, gives an indication of the requirements. Moliter,[170,171] in an early study on the application of ion propulsion for stationkeeping, identifies mission constraints that influence the engine system design and evaluates the tradeoffs among critical mission and system parameters such as attitude accuracy, average power utilization, energy storage requirement, duty cycles, thrust itervals, thrust level, and satellite mass and moment of inertia. Tom and Kalensher[172]

Table 3 Perturbation and velocity requirements

Cause of perturbation	Effects on satellite	Cost of correction expressed in "velocity increment ΔV" (ΔV= velocity variation that has to be imparted to satellite to cancel draft)
Radiation pressure Error of alignment of thrust vector with center of gravity Spurious impulses	Attitude drift (rotation around the center of gravity)	< 1m/s/yr (value usually given: 3 m/s, over 5 yr)
Triaxiality of Earth (lack of sphericity, leading to variations in terrestrial gravity potential)	Drift in longitude (East-West) toward the nearest stable point (75°E or 105°W) at the rate of 0.44 deg/day (Max)	5.5 m/s/yr
Solar radiation pressure	In-plane perturbations Increase of eccentricity	Function of satellite A/m, surface properties, and orientation
Moon-Sun attraction	Drift in latitude (North-South) of the orbital plane with respect to the equatorial plane, from 0.75 to 0.95 deg/yr, depending upon the value of lunar orbital inclination	Function of satellite pointing accuracy (alignment), of the order of 50 m/s/yr for a pointing accuracy of approximately 0.15 deg for the corrections applied in the immediate vicinity of each node (optimum operation from propellant consumption stand point)

investigate the motion of a synchronous satellite under the influence of a continuous radial microthruster, which is mechanically vectored for both attitude and orbit control. (Such a satellite will move in a synchronous orbit a little different from the normal one.) Balsam and Anzel[173] develop a basic mathematical model necessary for minimizing the motion in latitude and longitude. With a set of vector elements, they represent the motion as a superposition of many periodic terms, thus permitting the separation of higher-frequency terms from osculating elements. This is helpful in designing a control law. An automatic, self-contained stationkeeping system (eliminating the necessity of ground observations and orbit determination) is suggested by Braga Illa.[174] This has been successfully adopted for LES-6[154] and LES-8/9.[155] A closely related problem of orbit determination, accounting for stationkeeping forces, is studied by Lubowe[175] using a Kalman filter. Ostrander[176] analyzes the problem of fine control, requiring systems to overcome even the short periodic luni-solar perturbations. Flury[35] extends the study to the problem of removing long-period perturbations and gives useful plots for evolving a fuel-optimal strategy. Hunziker,[177] while evolving the maximum lifetime law for a low-thrust gravity-stabilized satellite, shows that even small initial errors in position and velocity can cause long-term longitudinal errors, requiring additional fuel consumption. This, therefore, calls for better modeling and more accurate orbit determination. Kamel et al.[178] arrive at somewhat similar conclusion for the two-maneuver East-West stationkeeping when they include the effects of luni-solar attraction in their model. It is found that the relative position of the Sun and Moon plays an important role in deciding the initial semimajor axis for the minimum-effort correction. In a series of recent investigations, Kaplan[135,150,179] considers various design and operational aspects of an all-electric system (i.e., without any other control or stabilizing force such as spinning, momentum wheel, etc.) for orbit and attitude control. It is concluded that three orthogonal components of control torque and continuous yaw sensing are required to insure a stable automatic control. The effects of a radial component of thrust indicate that alternate use of different North-South thrusters must be employed if they are canted away from the pitch axis. This is an important point to be noted for the application of electric thrusters for inclination control.

Stationkeeping of geosynchronous satellites can be treated as a process of transfer between two orbits very close together. As such numerous studies on optimization of impulsive and continuous-thrust trajectories become directly applicable, the vast amount of literature on the subject since the pioneering study by Hohmann[180] in 1925 is well reviewed in the excellent surveys by Leitmann,[181] Lawden,[182] Grodzovskii et al.,[183] Pitkin,[184] Edelbaum,[185] Marchal,[186-188] Robinson,[189] Gobetz and Doll,[190] Miele,[191] and Bell.[192] A few studies dealing specifically with the problem of optimal positioning and stationkeeping of geosynchronous satellites should be mentioned here. In an early study, Burkhart and Smith[193] apply dynamic programming to the orbital control process and find continuous correction to be better than discrete step system from the standpoint of accuracy, fuel economy, and acceleration level during correction. Than[194] develops an analytical solution for optimal positioning and stationkeeping. The problem of optimal transfer, apogee impulse, and positioning is analyzed by Biausse et al.[195,196] Gruver and Engerbach,[197] and Hughes.[198] The optimal low thrust ascent employing solar electric propulsion has also been receiving considerable attention lately. Some of the recent studies[199-201] dealing with minimization of time of ascent should be useful in evolving optimal stationkeeping strategies using microthrusters, too.

At the time of writing this survey, a conference on Attitude and Orbit Control Systems was being conducted by the European Space Agency. A large number of papers[202-210] presented there deal with the stationkeeping systems of the European satellites. These should be of interest to the readers.

C. Semipassive Orbit Control

The review of the active systems suggests a trend toward the development and use of lower-thrust chemical systems and more powerful efficient electrical thrusters for a gentle long-life reliable accurate control of the fragile spacecraft in the offing. The power and weight limitations and vagaries of the propulsive devices do make the problem rather complicated, requiring an advancement on a broad front of technology. A possibility of exploiting the disturbing environmental forces to advantage has attracted the attention of several investigators.[211,212] Of the three main sources of orbital perturbations for geosynchronous satellites, the use of solar radiation pressure has been considered most for orbital corrections. In an early study, Garwin[213] proposed a "solar sail" for space propulsion. Buckingham et al.[214] suggest the use of different reflective and absorptive coatings to produce a net force over a circular orbit to maintain the spacing between several passive near-Earth communication satellites. The method works well for near-ecliptic orbits; however, in resonant bands it fails. Miller et al.[215] consider the examples of hollow thin-skinned lenticular-shaped satellite and a plain "sail" structure. Two simple control procedures for programming the orientation of the surfaces to generate suitable stationkeeping forces are offered.[216] Black et al.[217] study three types of sails for orbital control of a geostationary satellite. It is found that a panel that has both sides reflective and rotates about the North-South direction with a 12-h period requires 1.47 m^2 for a 100-kg satellite for East-West stationkeeping at the longitude where the demand is maximum. Crocker[218] presents simulation details of such a controller. Shrivastava and Rajasingh[219] establish simple near-optimum control laws through parametric optimization for maximizing the corrections in the orbital elements. The results should be useful in optimal positioning and stationkeeping. Extending the concept of AEROSOL[220] for attitude control, Shrivastava et al.[221] evolve "SPACTS," a semipassive attitude-control and trajectory stabilization system employing SRP. Three schemes for East-West stationkeeping are considered. The best of the three reduces the demand on area-to-mass ratio to 1.2 m^2/100 kg. Grinevitskaia and Poliakhova[222] solve the equations of motion of a satellite with a sail moving in an arbitrary orbit. The study of the SRP effects on CTS leads Modi et al.[114] to find ways of reducing the influence by a slight maneuver of the solar panels. They conclude that even a maneuverability of $\pm$ 2 deg from the nominal Sun orientation can reduce the perturbations significantly. This finding is important for many current spacecraft being designed with large solar arrays. The study is followed up by Vander Ha and Modi,[223,224] who analyze the long-term behavior using a two-variable expansion procedure.

The preceding studies establish the feasibility of using solar radiation pressure for at least initial positioning, East-West stationkeeping, attitude control, and as a possible backup. A hybridization with the active systems also may improve their accuracy, duty cycle, and lifetime. Of course, there are several problems, which also possibly explain why these simple concepts have not been tried so far. Some of these are 1) control-structure interaction problems, which have been of great concern lately.[225-228] (enhanced because of the increased size of appendages); 2) associated problem of spacecraft charging[229]; 3) thermal deformations; 4) the problem of reliability due to moving parts, bearings, surface degradation, etc. (unfurling of large appendages also may pose problems in transient); 5) accuracy of orbit determination in the presence of the nonlinear controller; and 6) development of ultralightweight materials, usable for prolonged operations in the geostationary orbit environment. Although these are serious problems, it may not be wise to reject the ideas, especially in view of the concepts of the Satellite Solar Power Station (SSPS)[28,29] employing very large solar panels at the geostationary orbits for power generation.

IV. Concluding Remarks

A few papers and reports out of a vast amount of literature on the orbital perturbations and stationkeeping of geosynchronous communication satellites have been briefly reviewed to highlight the important aspects. No attempt has been made either to present any detailed analysis or to describe any system. The literature cited, especially a number of survey papers, amply fill this gap. The purpose here has been mainly to help the beginners and nonspecialists get an overview of the excitements and problems involved in the field, and to present the hard-pressed practicing engineers with at least a brief list of useful references. Many challenging problems emerge, some of which are: 1) further developments of simple mathematical tools to analyze perturbations, perhaps in totality; 2) analysis of solar radiation pressure on larger systems, such as SSPS, accounting for other perturbations and shadow effects, and identification of best launch conditions to minimize the control efforts; 3) study of the effect of charging and thermal deformations and thus altered SRP loading on the orbital behavior of large spacecrafts, 4) development of optimal total control systems for attitude and orbit correction; 5) further studies on autonomous control systems for the possible use of satellites for navigation and as a part of a tracking network; 6) more rigorous stability analysis of the systems; 7) more attention to a possible hybrid system of chemical/electric thrusters and also active/semipassive devices, with studies on optimal sharing of control; 8) more analysis of control-structure interaction problems, especially during the initial acquisition phase; 9) extensive error analyses and failure-mode studies, leading to useful design charts and criteria; 10) development of efficient simple tools for accurate orbit determination, accounting for perturbations and the control forces, especially for autonomous systems. (many techniques in the optimal control theory appear to be usable); 11) development of optimal systems suitable for Shuttle-based launches; 12) exploration of other environmental anomalies for possible semipassive control; and 13) testing of some of the semipassive concepts in flight (apparently some are planned now).

The list is by no means exhaustive. Let us hope that our endeavours, however abstract, unrealistic, and fancy they may appear today, will someday benefit even the poorest of the masses in the remote villages in the not-so-privileged nations.[230] In the words of United Nations Secretary General Waldheim,[231] "...space developments can go far in unifying mankind..."

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